ELE 459 : Digital Control Systems Lab #1

Design, Simulation, Implementation of an Analog Position Control System

Noah Johnson

Week 1 - *Derivation of Hardware Constants*
Week 2 - *Simulation of Phase Lead Compensator System*
Week 3 - *Implementation and Analysis of Analog Position Control System*

I. Summary

The goals of this lab was broken up by week. In the first week, the goal was to utilize Matlab and Simulink in conjunction with hardware to obtain a transfer function for the given motor-driven positioning system. During the second week, the experimentally obtained values were utilized to design a theoretical analog control system that was optimized for settling time while considering several hardware constraints. These constraints were a maximum plant input of 5 volts, and a maximum plant output of 3 position volts. Then, in the third week, the theoretical system was implemented in Simulink to approximate a real-time analog control system.

II. Results

Using Simulink and the motor driven positioning as previously described, step inputs of various heights (referred to as 'H' with units volts) were sent to the motor and the position output was recorded and plotted over time. Using plots for H = 1,2,3 Volts, α and β[1] values were found. The derived value for α was 26.6667 and the value for β was 133.33.

These values yielded the following transfer function for the system plant:

$$Y(s) = \frac{133.33}{s+26.6667} \qquad (2.1)$$

In the second week these derived values were used to design a phase lead compensator system for controlling the position of the cart. The constraints for the design were (1), the plant output could not exceed three pos-volts (no overshoot) and (2), the plant input could not exceed five volts. The plant gain factor β remained the same for this portion of the experiment, as this was a hardware constant.

The constant C_1 was chosen to cancel the plant pole at α=26.6667, such that C_1 = 26.6667. The constant f, which is a pure multiplicative factor for the compensator gain factor, was set to 1.5, since it was found that this reduced the settling time dramatically while having minimal effect on the plant input voltage. Then the compensator pole was tuned to minimize settling time without incurring overshoot, reaching a final value of 38.1. K and K_f (the gain factors for the compensator and prefilter, respectively) are dependent upon the other system constants and were not directly set. The prefilter gain K_f was 16.3672 and the compensator gain was 4.0184.

[1] pole location and transfer function gain factor, respectively

The overall transfer functions for the compensator and prefilter as implemented are as follows:

$$P(s) = \frac{16.3672}{s+16.3672} \tag{2.2}$$

$$C(s) = \frac{4.0184\cdot(s+26.6667)}{(s+32.7345)} \tag{2.3}$$

The final phase lead compensator system is 3rd order, and has poles at $s = 32.7345$, $s = 0$, and $s = 32.7345 \pm j\omega$ where $j\omega \ll \mathrm{Re}\{s\}$.

Finally in the third week the new control system was implemented on the motor driven cart system and utilized. This design allowed for simpler input, and a very short settling time, due to the degree of optimization during the previous week. However, the constants were highly optimized so there was just barely no overshoot, and the acceleration and deceleration of the cart was rather extreme, to the point where the entire system slightly moved on the table.

III. Equipment List

A. Hardware Components
   1. PC Tower running Windows Vista
      a) Software includes: MATLAB 2017a, Simulink 2017a
   2. Power Amplifier
      a) Aerotech 4020 Linear Servo Amplifier
      b) Serial Number: EFA401
   3. Motor Driven Cart System
      a) Aerotech 1000DC Permanent Magnet Servo Motor
      b) Part Number : 1050-01-1000
      c) Serial Number : 53012
   4. Digital to Analog Converter
      a) Part Number : PCI-DAS1002

IV. Procedure (Week One)

The first week of the lab was dedicated to deriving a transfer function for the motor driven cart system. The first step in this process was constructing a simulink model that could be used to supply a simple step input of height *H* and of duration T_{step} seconds. This would be the input to the Aerotech motor system (hereafter referred to as “the plant”). This input, also known as the function *u(t)*, was fed through a Digital to Analog converter then through a power amplifier, and finally to the plant.

This same plant also contains a rotary encoder assembly for tracking of the cart’s position along the track. The output from the rotary encoder on the plant was fed back into the simulink model using an analog to digital converter. This was then differentiated to obtain plant output velocity (as depicted in Figure 4.1) by the known kinematic relation:

$$v(t) = \frac{ds(t)}{dt} \tag{4.1}$$

In order to gather data, the Simulink real time windows targeting tool was used. First, the power amplifier was switched on, and the cart was reset to the safety position on the track. This was the point at which no input would have any effect on the position of the cart (this was to prevent any unexpected spikes in voltage from causing any danger to the equipment or the user). Then the simulink model was run once to ensure that there was no residual voltage in the line between the power amplifier and the plant. Then, the cart was moved out of the dead zone and the power amplifier was connected. The simulation file was then run on the hardware and the cart moved.

This velocity value was collected and graphed (see Figure 5.3) with respect to time for each of several trials, for differing values of H between 0 and 3 volts. The upper limit of 3 volts was a given constraint for safe operation of the plant. Any higher values would cause an acceleration that would not stabilize before the end of the track. Therefore the data would be useless, and the danger to both the equipment and the user would not be acceptable.

The settling time was then estimated, and the first of the two constants, the plant pole location (referred to as "α") was found using (5.1) - (5.7). From there, the plant gain factor (referred to as "β") could also be derived using the same equations. These values were derived for values of H = 1,2, and 3 Volts, and were averaged for a more robust value. This concluded the first week of the experiment.

V. Data and Calculations (Week One)

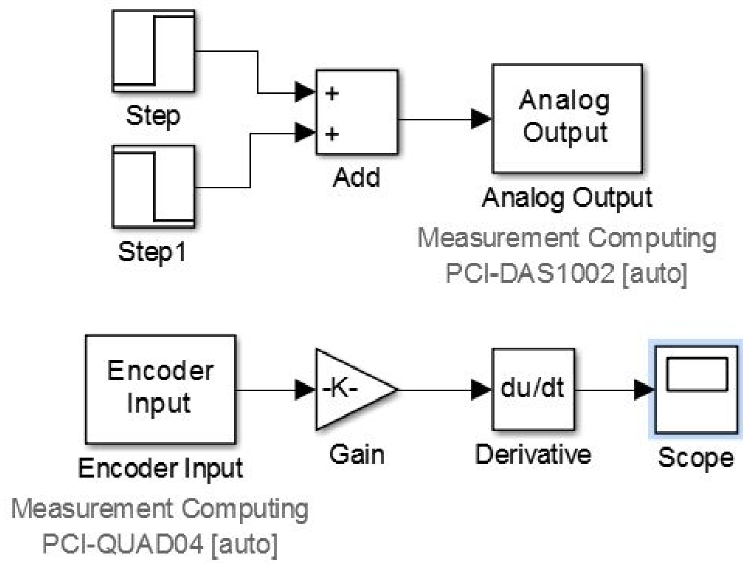

Figure 5.1 - *Simulink diagram for generation of step impulse and collection of data pertaining to cart velocity as a function of time. Used during week 1 of Lab to derive alpha and beta values for the plant.*

First, consider Figure 5.2, where $Y(s) = \frac{\beta}{s+\alpha}$:

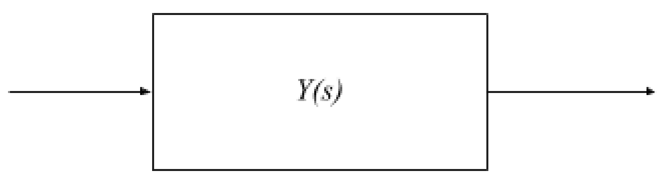

Figure 5.2 *- A generic system block with first-order transfer function Y(s)*

The step response of this system is then[2]:

$$y(t) = H \cdot \frac{\beta}{\alpha}(1 - e^{-\alpha t}) \quad (5.1)$$

[2] Observable by Laplace Transform

It then follows that the steady state reached as $t \to \infty$ is given by:

$$V_{SS} = H \cdot \frac{\beta}{\alpha} \quad (5.2)$$

Define the constant τ such that:

$$y(\tau) = 0.63 \cdot V_{ss} = 0.63 \cdot \left(H \cdot \frac{\beta}{\alpha}\right) \quad (5.3)$$

It is observable that $\tau = e^{-1}$, and that:

$$\alpha = \frac{1}{\tau} \quad (5.4)$$

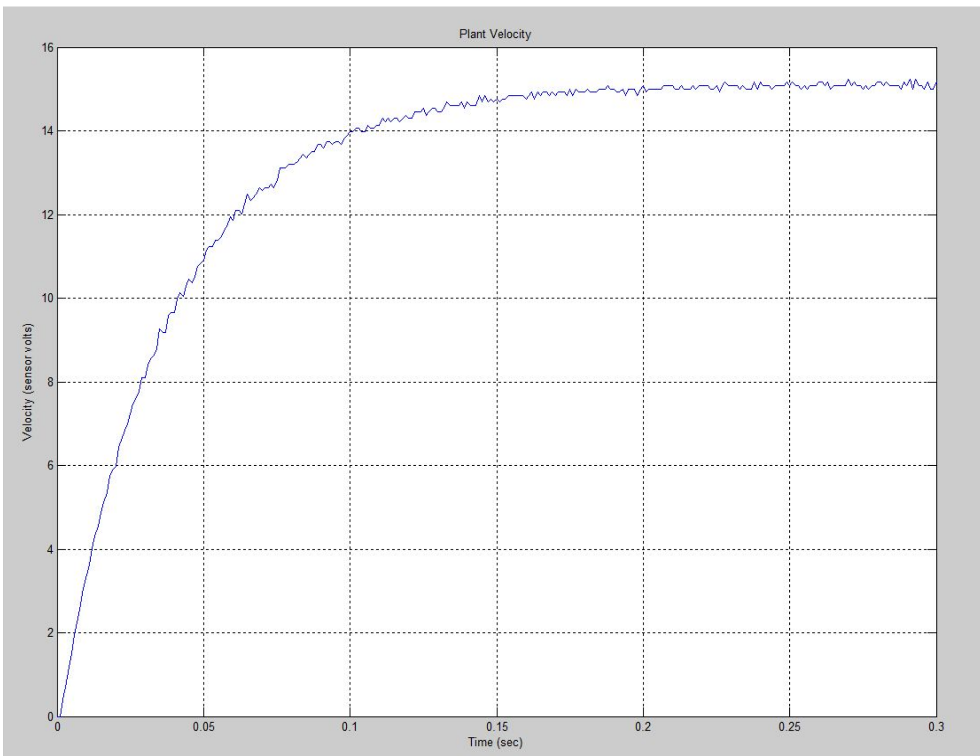

Figure 5.3 - *Matlab plot of the simple first week system, for H = 3, and* T_{step} *= 300 ms. Note that the cart velocity stabilizes at 15 Volts at time* T_{step} *= 250 ms.*

Applying this formula to the data in Figure 5.3:

$$\begin{aligned} \tau &= \{t \mid y(t) = 9.45\} \quad (5.5) \\ \Rightarrow y(t &= 0.0375) = 9.45 \\ \Rightarrow \tau &= 0.0375 \\ \alpha &= 26.66 \end{aligned}$$

By applying this same formula to tests for H = 1, 2, and 3 Volts a reliable average can be derived:

$$\widehat{\alpha} = \frac{\frac{1}{0.375}+\frac{1}{0.375}+\frac{1}{0.375}}{3} = 26.667 \tag{5.6}$$

This same technique can be used to derive an average beta value. If we find the average ratio of V_{ss} and H we can use this equation:

$$\frac{\widehat{\beta}}{\widehat{\alpha}} = \frac{\frac{V_{ss1}}{H_1}+\frac{V_{ss2}}{H_2}+\frac{V_{ss3}}{H_3}}{3} \tag{5.7}$$
$$\Rightarrow \widehat{\beta} = 133.333$$

VI. Procedure (Week Two)

During the second week of this experiment, the previously obtained plant constants were utilized to build a more robust position control model. The goal was to reduce system settling time, T_s ,as much as possible without incurring any overshoot in the plant output voltage, *y(t),* or exceeding the previously established 3 Volt maximum on the plant input, *u(t)*. This was realized using the closed system feedback model, using a compensator and prefilter, as seen in Figure 7.1. This overall system is known as a "Phase Lead Compensator"

The simulation procedure was as follows. First Simulink and Matlab were opened and the necessary constants and simulation blocks were added to the workspace. Then a first simulation run was done to establish a starting point. Then, the variables f and α^* were tweaked to optimize the system. Final values and graphs for these values can be found in section 7.

VII. Data and Calculations (Week Two)

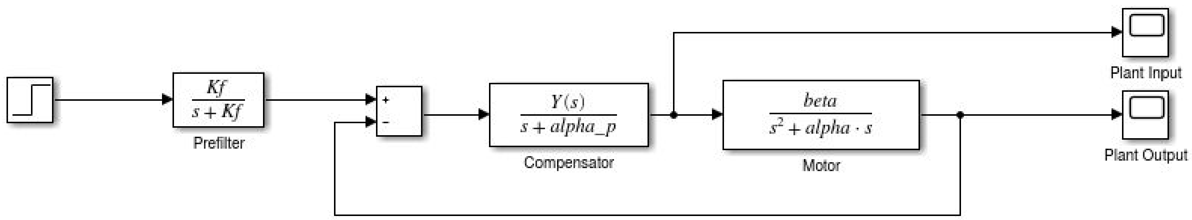

Figure 7.1 *- Simulink Model for Phase Lead Compensator System as implemented.*

Consider the system model shown in Figure 7.1. Define the following transfer functions for the Prefilter, Compensator, and Motor[3], respectively:

$$P(s) = \frac{K_f}{s+K_f},\ C(s) = K \cdot \left(\frac{s+C_1}{s+\alpha^*}\right), Y(s) = \frac{\beta}{s(s+\alpha)} \tag{7.1}$$

If the feedforward path of the feedback loop G(s) is considered, the constant C_1 can be choses to be equal to α, thus cancelling the plant pole.

The effective feedforward path transfer function is then:

$$G(s) = \left(\frac{K}{s+\alpha^*}\right) \cdot \frac{\beta}{s} \tag{7.2}$$
$$G(s) = \frac{K\beta}{s^2+s\alpha^*}$$

It follows that the overall feedback transfer function F(s) is:

$$F(s) = \frac{G(s)}{1+G(s)} \tag{7.3}$$
$$F(s) = \frac{\frac{K\beta}{s^2+s\alpha^*}}{1+\frac{K\beta}{s^2+s\alpha^*}} = \frac{K\beta}{s^2+s\alpha^*+K\beta}$$
$$F(s) = \frac{133.33K}{s^2+s\alpha^*+133.33K}$$

[3] The differentiator block from the previous diagram (Figure 5.1) has been integrated into the plant for simplicity.

It is observable from (7.3) that the feedback system is 2nd order. In order to optimize the settling time of this system, the poles of the system must be equal. To achieve this, the roots of the denominator polynomial must be found using the quadratic formula:

$$s^2 + s\alpha^* + 133.33K = 0 \tag{7.4}$$

Note that the gain factor of the compensator K is given by:

$$K = \frac{(\alpha^*)^2}{4\beta} \tag{7.5}$$

Therefore (7.4) becomes:

$$s^2 + s\alpha^* + \frac{(\alpha^*)^2}{4} = 0 \tag{7.6}$$

By applying the quadratic formula, the optimal double pole location of the feedback system can be found at:

$$s = \frac{-\alpha^*}{2} \tag{7.7}$$

However, the addition of a prefilter causes this system to no longer be 2nd order, but instead 3rd order. This means that (7.7) will yield a double pole that is not optimal for the overall system settling time. It is a known property of 3rd order control systems that the poles of the system may have complex parts without incurring overshoot, so long as *Re{pole}* >> *Im{pole}*. Therefore, the optimal solution can be found by scaling K such that the poles of the feedback portion of the system take on some small imaginary part without changing their real part. The chosen scale factor f was set to 2 after much trial and error. This factor was found to minimize the settling time of the system without incurring overshoot. The overall formula formula for K then becomes:

$$K = f \cdot \frac{(\alpha^*)^2}{4\beta} \tag{7.8}$$

Recall that the prefilter has a purely real pole at $s = K_f$, by (7.1a). Recall also that the idealized 3rd order system solution has a triple pole. Therefore, we want the pole of the prefilter to be equal to the real part of the double complex pole, which was found in (7.7).

It follows then that:

$$K_f = \frac{-\alpha^*}{2} \tag{7.9}$$

As with the scale factor *f*, the numerical value for α^* was derived through tuning of the simulation using a MATLAB local optimization algorithm. The optimal value for α^* was found to be 32.7345. With these simulation parameters, the maximum input plant voltage was 5 volts, and the settling time was 283.3 ms. See Figure 7.3 for Matlab console output of these values. Graphs of the simulated system can be found in Figure 7.2. The final derived transfer functions for the prefilter and compensator are as follows:

$$P(s) = \frac{16.3672}{s+16.3672} \tag{7.10}$$

$$C(s) = \frac{4.0184 \cdot (s+26.6667)}{(s+32.7345)} \tag{7.11}$$

This concluded Week 2 of the experiment.

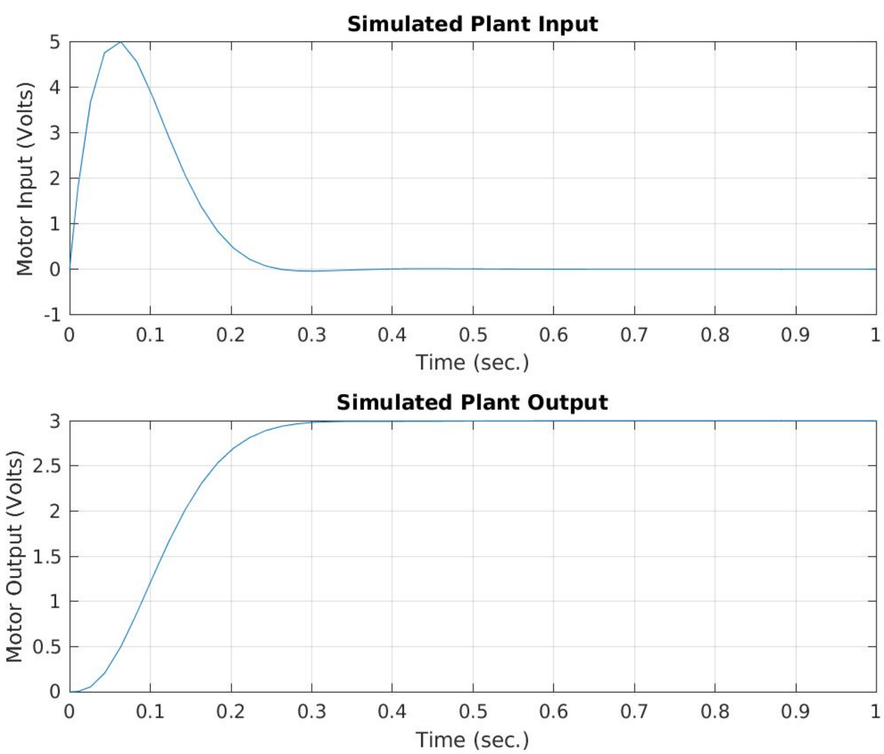

Figure 7.2 *- Input and Output of simulated 3rd order system solution, with simulation parameters* α^**=32.7345, and f = 2.*

```
max_input =

    5.0000


Ts =

    0.2833
```

Figure 7.3 *- MATLAB Console Output of max simulated plant input voltage and 1% system settling time obtained from Figure 7.1, 7.2*

VIII. Procedure (Week Three)

During the third week of the experiment, the phase lead compensator model derived in the previous week was implemented and ran on the Aerotech system. The simulink diagram from week one was modified to include the transfer functions that were added in week two, and the model was tested in the same way as during week one, with a step input of H = 3.[4] Relevant data and plots can be found in section 9.

IX. Data and Calculations (Week Three)

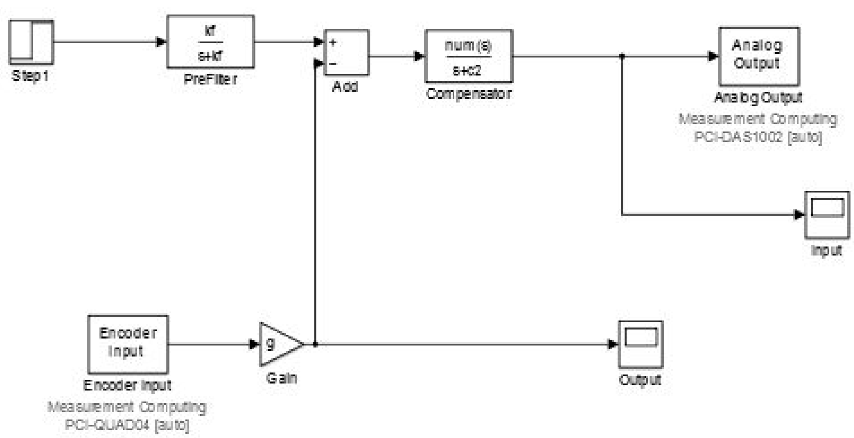

Figure 9.1 *- Simulink Diagram of Phase Lead Compensator system as implemented during week three*

[4] See section IV for Week One procedure and Safety guidelines.

Consider the diagram shown in Figure 9.1. It is known that the output of the Encoder is in divisions of 1 Volt per 20 rad of rotation. Therefore, there is a gain factor of $\frac{\pi}{40000}$ applied so that the output is in volts.

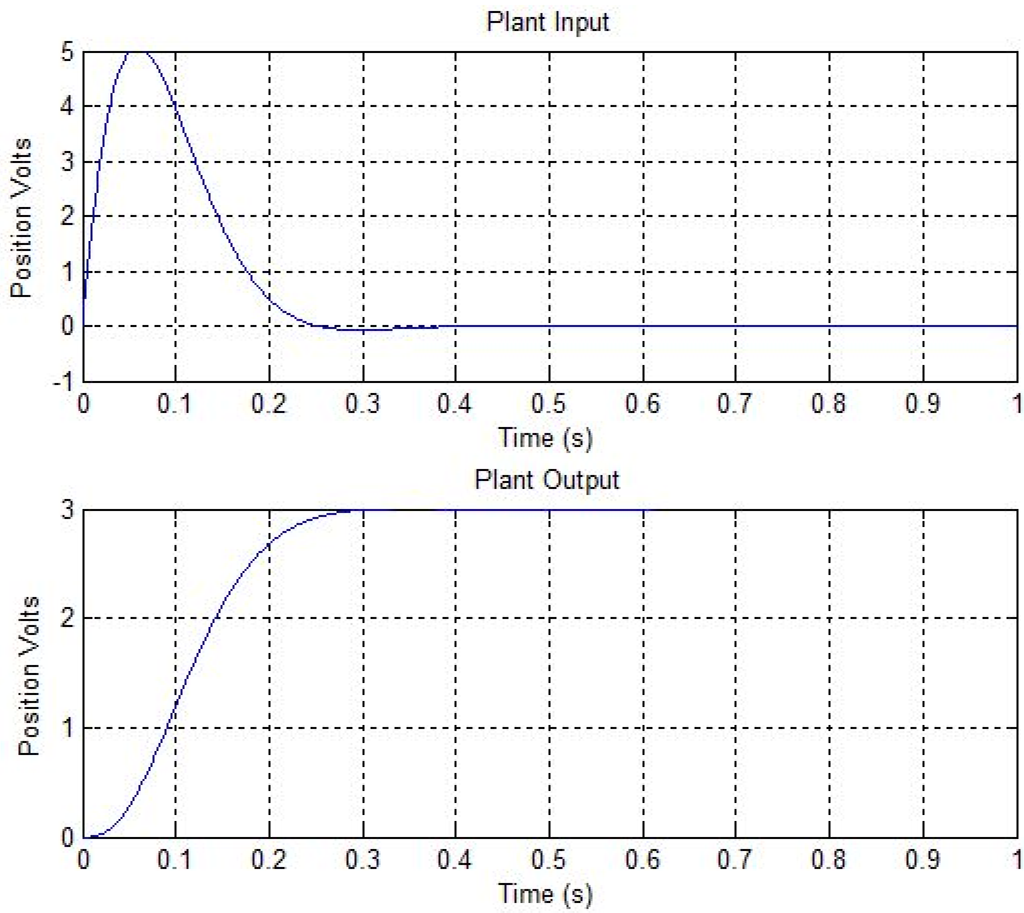

Figure 9.2 *- Experimental Plant Input and Output for Phase Lead Compensator System*

Consider the graphs shown in Figure 9.2. By observation, the settling time of the system is slightly less than 0.3 seconds, and thus matches the simulation very closely.

This concludes Lab 1.